Macroscopic Reality in Quantum Mechanics; Origin and Dissipation

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Abstract

We study the connection between dissipation and reality in macroscopic quantum systems. We present the following scenario; if we consider the dynamics of a 'partial' wave function, the dissipation is represented as a nonlocal term and it causes destructive interference to suppress the quantum fluctuation. Using the variational method, we confirm that this dissipation term is a reasonable extension of the standard (Schrödinger) description for isolated systems, from which we also derive the classical action. Consequently, in macroscopic systems, the states whose time-integrated dissipation takes an extreme value come true. This description, which is consistent with our sense of reality, coexists with the usual linear-time-dependent description.

1 Introduction

1.1 Motivation

Early in the 20th century classical mechanics was abandoned and giving way to quantum theory on the simplest and the most fundamental systems, i.e. photons, electrons and atoms. After that, although the accumulation of diverse experimental results has made, we have never encountered its limitation. Far from it, its foundation as a fundamental theory of physics is being more solidified.

The essence of quantum theory is often expressed as noncommutativity or waviness. In non-relativistic region, this leads quantum mechanics and we have some equivalent but different styles of mathematical representations. In addition, since each of these representations has the ability to give *autonomous* models for various situations and both the number of elements and the expanse of the system do not restrict these models themselves, we recognize that there is an obvious *continuity* between micro- and macroscopic systems. In fact, this continuity has brought great success on our understanding of the material world and applications. Therefore it is natural that we should expect quantum mechanics to make radical change in everyone's worldview.

Despite the expectation, something seems to limit the applicability of quantum concepts or the continuity of the description. Our daily experience is itself a common example.

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One of the mathematical expressions, so-called the Schrödinger equation, declares that the state of a system is a kind of linear wave. Therefore, in the same way as all the other linear waves, any superposition of states can be allowed. In fact, this property is absolutely important to understand the microscopic origin of our material world. However, when we extend it to macroscopic phenomena, it is sharply confronted with our everyday experience; we never observe directly such wavy features in our surroundings. On the other hand, classical mechanics holds with amazing accuracy, which can't be expected from quantum mechanics. Therefore, it occurs to us that quantum features, i.e. fluctuation or waviness, seem to be perfectly suppressed for some reason.

On the other hand, we also encounter vivid examples in laboratories. For example, on a fluorescent screen or in a bubble chamber, even a single elementary 'particle' isn't observed as a wave, i.e. a superposition of position states, but like a material point. Although it is contradictory to the Schrödinger equation, we thus cannot abandon the old-fashioned statement that an elementary particle is a substance without volume. Not only in a position of particle, but in any physical quantity we always fail to observe a quantum superposition or waviness. The simple question occurs to us; does this mean that the *watch* intervenes the mathematical description in which the whole world should be autonomously evolving with time? This question is regarded to be reasonable and often called the measurement problem, since the gap becomes acute when we intend to observe microscopic properties directly. Although many researches (for example in [1]) have already been made, it stays unsolved to explain the origin of the descriptive gap between quantum mechanics and our direct experience.

In contrast to its appearance, it is difficult to resolve the above problems into a well-defined style since quantum mechanics seems to provide an autonomous model for any kind of situation[2]. In this paper, we particularly focus on the origin of the classicality and the reality in macroscopic systems.[1]

1.2 Strategy

Throughout our study, we consider the meaning of the fact that a system can be described by a single wave function.

In §2.1, we consider to divide the system into two parts. Then we find that the integration of the external degrees of freedom naturally leads the nonlocal dissipation term. In §2.2, the dynamics of the disappearance of superposition states is shown, where we find the conditions for the stability of partial systems. It is also shown that two descriptions coexist; one is that a whole system evolves linearly with time, and the other is that partial systems are interacting each other. In §3.1, it is verified that the dissipation term is the natural extension of the standard Schrödinger equation. In §3.2, we obtain the conservation laws for physical quantities and the classical action.

[1] We use the word 'classicality' to refer to the fact that the physical quantities behave like those of classical mechanics and 'macroscopic reality' to refer to our sense of reality, which is based on both the classicality and the stability of macroscopic systems.

2 Relation between macroscopic reality and dissipation

Usually interacting degrees of freedom are mixed. In some cases, however, the wave function of a partial system has an important role. For example, a Schrödinger equation with an external field is nothing more than that we obtain by fixing external degrees of freedom and integrating them. Let us consider the conditions that make it possible to describe a partial system with a single wave function. Then it will be confirmed that a nonlocal term is added to the Schrödinger equation when the system dynamically interacts with outside.

2.1 Schrödinger equation for a partial system

In this subsection, we make a preliminary discussion about the contact of macroscopic quantum systems.

Let us start from the familiar Schrödinger equation satisfied by the wave function of a whole system[2];

$$[-i\hbar\frac{\partial}{\partial t}+\hat{H}]\Phi_0=0, \tag{1}$$

where

$$\hat{H}=-\sum_{1\le i\le N}\frac{\hbar^2}{2m_i}\nabla_i^2+\sum_{1\le i<j\le N}\frac{q_iq_j}{|\mathbf{r}_i-\mathbf{r}_j|}. \tag{2}$$

To survey the contact of two systems we try an approximate solution, $\Phi=\varphi(\mathbf{r})\Psi(\mathbf{R})$.[3] Each of the wave function is normalized. We can also separate the Hamiltonian as $\hat{H}=\hat{h}_\varphi(\mathbf{r})+\hat{h}_{int}+\hat{h}_\Psi(\mathbf{R})$, where

$$\hat{h}_\varphi(\mathbf{r})=-\sum_{1\le i\le m}\frac{\hbar^2}{2m_i}\nabla_i^2+\sum_{1\le i<j\le m}\frac{q_iq_j}{|\mathbf{r}_i-\mathbf{r}_j|}, \tag{3}$$

$$\hat{h}_{int}=\sum_{1\le i\le m<j\le N}\frac{q_iq_j}{|\mathbf{r}_i-\mathbf{r}_j|}. \tag{4}$$

This is a kind of mean field approximation, which is usually valid under weak interaction. For the present, we go forward assuming that this description holds at least at the beginning of the contact.

To obtain the wave equation for the partial system φ, we multiply (1) by Ψ^* from the left side, and integrate it over $\mathbf{R}$ under the condition of $\int d\mathbf{R}|\Psi|^2=1$, where $d\mathbf{R}=d\mathbf{r}_{m+1}\cdots d\mathbf{r}_N$. Then, we get

$$[-i\hbar\frac{\partial}{\partial t}+\hat{h}-\lambda(t)]\varphi=0, \tag{5}$$

$$\hat{h}=\hat{h}_\varphi(\mathbf{r})+V(\mathbf{r},t), \tag{6}$$

[2]In this paper, we neglect spin and quantum statistics for simplicity, and take account of only Coulomb interaction.

[3]$\varphi(\mathbf{r},t)=\varphi(\mathbf{r}_1,\cdots,\mathbf{r}_m,t), \Psi(\mathbf{R},t)=\Psi(\mathbf{r}_{m+1},\cdots,\mathbf{r}_N,t)$.

where

$$\lambda(t) = \int_{-\infty}^{\infty} d\mathbf{R}\Psi^*(\mathbf{R},t)[i\hbar\frac{\partial}{\partial t} - \hat{h}_\Psi(\mathbf{R})]\Psi(\mathbf{R},t). \tag{7}$$

and the 'external field' $V(\mathbf{r},t) = \int d\mathbf{R}\hat{h}_{int}|\Psi|^2$. If we combine equation (5) with its complex conjugate, we get the equation for the current conservation and find λ to be real. When V is independent of φ and $\lambda(t) = 0$, equation (5) is nothing but the Schrödinger equation for an isolated system. We, in principle, have to solve Ψ and φ at the same time. In this sense, V and λ have non-linearity.[4] Although such non-linearity is a source of interest, for our purpose, it is more important that equation (5) is a pseudo-linear form with the eigenvalue λ, i.e.

$$[\hat{L} - \lambda(t)]\varphi = 0, \tag{8}$$

where $\hat{L} = -i\hbar\frac{\partial}{\partial t} + \hat{h}$. Therefore it is suggested that macroscopic states are classified under the 'energy transfer' $\lambda(t)$, which we call *dissipation* (as is seen later in eq. (27) in §3.2). We notice that λ has a nonlocal influence on the configuration space.

Since $\Phi = \varphi\Psi$ is an approximate solution, the interaction makes $[-i\hbar\frac{\partial}{\partial t} + \hat{H}]\Phi \neq 0$ with the passage of time. In other words, the idea of 'partial system' seems unstable. However, the standard quantum mechanics, which describes particles in an external field, teaches us the wide-ranging validity of this idea. In the next subsection we will investigate the origin of such stability of partial systems.

2.2 Emergence of a partial system

If φ is at a superposition state (in a secific basis), we naively predict that the interaction makes Ψ move to the corresponding superposition state. This prediction, however, is sharply confronted with our daily experience consisting of macroscopic quantum systems. We see below how the occurrence of a superposition is suppressed by dissipation.

We take up the state Φ_0 again, which is one of the exact solution of the Schrödinger equation (1). Here we try to divide this system into two partial systems, φ and Ψ, where we particularly consider the thermal contact of these systems. (Below, one can imagine Ψ as a heat bath for φ.) Although Φ_0 will be gradually away from the mean field solution $\Phi = \varphi\Psi$, (i) we assume that we can expand it with the mean field solutions, $\Phi_\nu = \varphi_\nu\Psi_\nu$;

$$\Phi_0 = \sum_\nu \alpha_\nu\Phi_\nu, \tag{9}$$

$$\alpha_\nu = \int_{-\infty}^{\infty} d\mathbf{r} \int_{-\infty}^{\infty} d\mathbf{R}\Phi_\nu^*\Phi_0. \tag{10}$$

To obtain the description for the partial system, we multiply equation (1) by $\sum_{\nu'} \alpha_{\nu'}^*\Psi_{\nu'}^*$ from the left side and integrate it over $\mathbf{R}$. We get

$$\sum_\nu |\alpha_\nu|^2[\hat{L} - \lambda_\nu(t)]\varphi_\nu = 0, \tag{11}$$

$$\hat{L} = -i\hbar\frac{\partial}{\partial t} + \hat{h}, \tag{12}$$

[4] V_φ and λ_φ may be more appropriate notations.

where $\lambda_\nu(t) = \int d\mathbf{R}\Psi^*_\nu[i\hbar\frac{\partial}{\partial t} - \hat{h}_\Psi]\Psi_\nu$ and $\hat{h} = \hat{h}_\varphi + V_\nu$. We have neglected the off-diagonal terms, $\int d\mathbf{R}\Psi^*_{\nu'}\cdots\Psi_\nu$, which are small enough in most systems with large degrees of freedom.

Now, to concentrate on the effect of the nonlocal term λ, we neglect the non-linearity and the time dependence in V. This is justified for the thermal contact. If we let ϕ_ν be the solution for the isolated case (i.e. $\hat{L}\phi_\nu = 0$), the solution for $[\hat{L} - \lambda_\nu]\varphi_\nu = 0$ is expressed as

$$\varphi_\nu = \phi_\nu e^{i\Lambda_\nu(t)/\hbar}, \tag{13}$$

where Λ_ν is the function that satisfies $\dot{\Lambda}_\nu = \lambda_\nu$. Although we cannot exactly define the partial system at this point, from looking at equation (11), it seems natural to define the partial wave function as

$$\varphi = \sum_\nu |\alpha_\nu|^2 \phi_\nu e^{i\Lambda_\nu(t)/\hbar}. \tag{14}$$

We impose two more conditions on $\Lambda_\nu(t)$; (ii) the members of $\{\Lambda_\nu\}$ are dense enough, and then (iii) the absolute value of Λ_ν strongly depends on ν by the unit of $\hbar$. To satisfy the former assumption, it is necessary that the degrees of freedom of Φ_0 is sufficiently large. The latter assumption is probably satisfied even in semi-macroscopic Φ_0 because of the large factor $k_B/\hbar \simeq 1.3\times10^{11}$[K/s] for the thermal fluctuation. Then, $e^{i\Lambda_\nu/\hbar}$ (with the weight factor $|\alpha_\nu|^2$) strongly oscillates with ν and causes destructive interference among the states. Therefore the ν's whose $\Lambda_\nu(t)$ has an extreme value mainly contribute in equation (11) and (14). Being the assumptions (i)-(iii) sufficiently satisfied, since only one ν_c survives, equation (11) becomes

$$[\hat{L} - \lambda_{\nu_c}]\varphi_{\nu_c} = 0, \tag{15}$$

and (14) becomes

$$\varphi \rightarrow \varphi_{\nu_c}. \tag{16}$$

Similarly, we obtain $\Psi \rightarrow \Psi_{\nu_c}$ (the same index ν_c is chosen because $\lambda + \lambda_\Psi = const.$ for static V.) Therefore we can regard the whole system as $\Phi = \varphi_{\nu_c}\Psi_{\nu_c}$.

Although the description Φ_0 still holds, Φ is the right description to correspond to our recognition. In other words, the truly quantum description Φ_0, whose degrees of freedom are mixed, is beyond our sense of reality. Even if such doubleness of the description is surprising, it becomes possible to make the concept of 'partial system' coexist with the unitary time evolution of quantum mechanics.[5]

Here we briefly comment on the measurement problem. For example, under strong Coulomb interaction one can expand the wave function with the coordinates at a specific

[5]Of course we can guess that Φ_0 may be the partial system of a larger system, i.e. the isolation of Φ_0 can be apparent (due to $\lambda_{\Phi_0} = 0$). We, however, did not mention this in order to have our discussion converge. For the same reason, we did not discuss the case that the way to divide Φ_0 into φ and Ψ is not unique.

time (according to Huygens' principle), and assign them indices ν's. (All of these localized states give extreme values to Λ, while the delocalized states disappear due to the interference.) If only one ν survives along the above scenario, it is probable that this process corresponds to the position measurement. We stress that our scenario is deterministic in the description Φ_0. Despite this, the destructive interference looks like acausal in the configuration space of the partial system φ. Only such nonlocal effect can describe the process $\varphi \to \varphi_{\nu_c}$ (see for example [3]).

We have to notice that Φ cannot be a permanent solution of any wave equation and equation (15) is also temporary. In other words, the quantum jump (transition) occurs corresponding to the change, $\nu_c \to \nu_{c'}$.[6] Macroscopically, the transition, $\nu_c \to \nu_{c'} \to \cdots \to \nu_{c''}$, causes 'thermal' energy transfer and 'friction'.

There is no denying the possibility that a superposition of different ν's survives when the assumptions (i)-(iii) are insufficiently satisfied. In common macroscopic objects, however, we can safely expect that the thermal fluctuation is always suppressing the quantum fluctuation. Therefore we assume $\Phi = \varphi\Psi$ in the next section.

3 Action and physical quantities

While we introduced $\lambda(t)$ by integrating the external degrees of freedom in §2, we here present the discussion in a more deductive manner. We study the conservation laws for macroscopic systems and verify the consistency of our approach.

From our daily experience, a macroscopic object, which justly consists of quanta, occupies a certain domain in space and time. Therefore it occurs to us that from a certain scalar, namely an action, we derive physical quantities as well as in classical mechanics. These quantities correspond to the expectation values in quantum mechanics, and the action should also produce the corresponding wave equation.

Although the friction discussed in §2.2 is important, we here concentrate on the quasistatic case, i.e. considering the term in which the wave equation holds.

3.1 Action principle

From the following action[7], we can develop all of the discussion.

$$S = \int_{t_2}^{t_2} dt \int_{-\infty}^{\infty} d\mathbf{r} \varphi^* \hat{L} \varphi, \tag{17}$$

$$\hat{L} = -i\hbar \frac{\partial}{\partial t} + \hat{h}, \tag{18}$$

where

$$\hat{h} = \hat{h}_\varphi(\mathbf{r}) + V(\mathbf{r}, t). \tag{19}$$

Under the norm condition $\int_{-\infty}^{\infty} d\mathbf{r} |\varphi|^2 = 1$, we require this action to satisfy the stationary condition in the $3m+1$ dimensional configuration space-time. For example, with $\delta\varphi^*(t_1) =$

[6] If the energy is transferred only by real photons, $\Lambda(t) = n(t)\hbar$.

[7] The action is also expressed as $S = \Lambda(t)\big|_{t_1}^{t_2}$, where Λ is the time integrated dissipation in §2.

$\delta\varphi^*(t_2) = 0$, the variation $\varphi^* + \delta\varphi^*$ gives the Schrödinger equation containing Lagrange multiplier $\lambda(t)$;

$$\frac{\delta}{\delta\varphi^*}\left[S - \int_{t_2}^{t_2} dt\lambda(t)\int_{-\infty}^{\infty} d\mathbf{r}|\varphi|^2\right] = [\hat{L} - \lambda(t)]\varphi = 0. \tag{20}$$

This wave equation is the same as equation (8) in §2 and includes the isolated case as $\lambda(t) = const.$ (Being V and λ real, the variation $\delta\arg\varphi$ gives the current conservation.)

It should be justified to ignore the non-linearity of V and λ in the above variation. To give a suggestion about this, we consider the total (mean field) action;

$$\mathcal{S} = \int dtd\mathbf{r}d\mathbf{R}\Phi^*[-i\hbar\frac{\partial}{\partial t} + \hat{H}]\Phi \tag{21}$$

$$= S + \int dtd\mathbf{R}\Psi^*[-i\hbar\frac{\partial}{\partial t} + \hat{h}_\Psi]\Psi. \tag{22}$$

We assign φ_α to φ and φ^*, as well as Ψ_β to Ψ and Ψ^*, for convenience. The variational parameters $\delta\varphi_\alpha$ and $\delta\Psi_\beta$ give four wave equations. Firstly we solve the equations $\delta\mathcal{S}/\delta\Psi_\beta = 0$ and obtain Ψ_β as functionals of φ_α. Substituting this, we obtain $\tilde{\mathcal{S}}$ as an *implicit* functional of φ_α. Let us express such variation as $d\varphi_\alpha$. Then the variation of this action is represented as

$$\frac{d\tilde{\mathcal{S}}}{d\varphi_\alpha} = \frac{\delta S}{\delta\varphi_\alpha} + \frac{\delta\tilde{\mathcal{S}}}{\delta\Psi_\beta}\frac{d\Psi_\beta}{d\varphi_\alpha} = 0. \tag{23}$$

We need to consider only the first term because the second term is zero.

3.2 Physical quantities and classical action

We know that the conservation laws and the symmetry of a system are closely connecting each other; symmetry operations guide to the relation between a wave function and physical quantities. To simplify the expression we show the case of one-body momentum and energy below.

It is obvious that the action doesn't change when the integral parameter x is shifted to $x + \Delta x$. Due to this shift, there are two types of change in the integral; one is caused from $\Delta\varphi = \frac{\partial\varphi}{\partial x}\Delta x$ and the other is from $\Delta V = \frac{\partial V}{\partial x}\Delta x$. We represent these contributions such as $\Delta S = \Delta_\varphi S + \Delta_V S$. Using the wave equation, the first term is transformed into $\Delta_\varphi S = \int_{-\infty}^{\infty} d\mathbf{r}\varphi^*\frac{\hbar}{i}\frac{\partial}{\partial x}\varphi\Big|_{t_1}^{t_2}\Delta x$, which is related to the conservation law because only the initial and the final state contribute to this. Of course this is what the orthodox quantum mechanics teaches us as the x component of the momentum. Therefore, if we use the notation $p_x(t) = \int_{-\infty}^{\infty} d\mathbf{r}\varphi^*\frac{\hbar}{i}\frac{\partial}{\partial x}\varphi$, we find

$$\frac{\Delta S}{\Delta x} = p_x\Big|_{t_1}^{t_2} + \int_{t_1}^{t_2} dt\int_{-\infty}^{\infty} d\mathbf{r}|\varphi|^2\frac{\partial}{\partial x}V = 0, \tag{24}$$

$$p_x\Big|_{t_1}^{t_2} = p_x(t_2) - p_x(t_1). \tag{25}$$

The time derivative of this equation gives Newton's second law;

$$\dot{\mathbf{p}} = -\int_{-\infty}^{\infty} d\mathbf{r}|\varphi|^2 \nabla V. \tag{26}$$

Similarly, starting from $\dfrac{\Delta S}{\Delta t} = \dot{\Lambda}(t)\Big|_{t_1}^{t_2} = \lambda(t)\Big|_{t_1}^{t_2}$, we obtain the conservation law for the energy;

$$\mathcal{E}\Big|_{t_1}^{t_2} = -\lambda(t)\Big|_{t_1}^{t_2} + \int_{t_1}^{t_2} dt \int_{-\infty}^{\infty} d\mathbf{r}|\varphi|^2 \frac{\partial}{\partial t} V, \tag{27}$$

$$\mathcal{E} \equiv \int_{-\infty}^{\infty} d\mathbf{r}\varphi^* i\hbar \frac{\partial}{\partial t}\varphi. \tag{28}$$

This corresponds to the first law of thermodynamics. $\lambda(t)\Big|_{t_1}^{t_2}$ is understood as the amount of heat to flow out.

These results are easily extended for N-body case. Finally we consider the case that the system can be described within the framework of classical mechanics by eliminating the degrees of freedom. Let us assume that the system consists of same kind of particles and pay attention only to the center-of-mass motion. Considering the translation of all the elements at the same time, we obtain the variation of the action as[8]

$$\delta S = \Delta_\varphi S \tag{29}$$

$$= \mathbf{p} \cdot \Delta \mathbf{q} - \mathcal{E}\Delta t, \tag{30}$$

where $\mathbf{p} =< \sum_i \hat{\mathbf{p}}_i >= \int d\mathbf{r}\varphi^* \frac{\hbar}{i} \sum_i \nabla_i \varphi$. Here we can interpret $(\Delta \mathbf{q}, \Delta t)$ as the infinitesimal path of the center-of-mass. We also assume that the internal state does not change; λ and the dispersion $< \sum_i \hat{\mathbf{p}}_i^2 > /N- < \sum_i \hat{\mathbf{p}}_i >^2 /N^2$ are constant. Moreover, using the approximation $\int d\mathbf{r}|\varphi|^2 V \simeq V(\mathbf{q})$, we obtain $\mathcal{E}(\mathbf{p}, \mathbf{q}) = \dfrac{\mathbf{p}^2}{2M} + V(\mathbf{q}) + const.$, where $M = Nm$ is the total mass. There remains only two degrees of freedom, i.e. the path and the momentum. Then we obtain the classical action;

$$S_{path}(\mathbf{p}, \mathbf{q}) = \int \Delta_\varphi S \tag{31}$$

$$= \int_{t_1}^{t_2} (\mathbf{p} \cdot \dot{\mathbf{q}} - \mathcal{E}(\mathbf{p}, \mathbf{q}))\Delta t. \tag{32}$$

$\delta S_{path}/\delta \mathbf{p} = 0$ and $\delta S_{path}/\delta \mathbf{q} = 0$ give $\mathbf{p} = M\dot{\mathbf{q}}$ and equation (26), respectively.

4 Conclusions

We have studied the connection between the classicality and the dissipation in order to derive the macroscopic reality from quantum mechanics. We presented the scenario, where the dissipation causes the destructive interference between superposition states. In other

[8] $\hat{h}_\varphi$ is invariant under this transformation.

words, in this case, the macroscopic system can be described as a aggregate of stable partial systems and it behaves like a classical system.

While we can understand a wave function as a matter wave in the configuration space (§3), the nonlocality of λ is also important when we consider the dynamics of the system (§2). This suggests the totality (namely, the imperfection of the 'partial system'), which has been often mentioned (for example in [1]) in the context of the measurement problem. As the result, the macroscopic reality consists of the processes (expressed by Φ) that make the action (namely, the time integrated dissipation) take an extreme value. Behind this, the unknowable linear description (Φ_0) exists.

If it is difficult, there is a possibility of checking the correctness of our scenario to use so-called 'macroscopic quantum systems', i.e. the collapse of a superposition can be observed by controlling the assumptions (ii) and (iii) in §2.2. The observation of such collapse has already been discussed (for example in [4]). Of course, by numerical calculation with proper approximation, we can confirm the validity of the assumptions (i)-(iii).

Although we found that there are two types of time-evolution in our macroscopic reality, i.e. the transition $\varphi_{\nu_c} \to \varphi_{\nu_{c'}}$ and the linear-time dependent evolution, the connection between the former time-asymmetric process and the second law of thermodynamics must be studied. We recognize lack of quantitative evaluation and applications for concrete cases throughout this study. Critical examination of this rough sketch is necessary.

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